WS 2016/17

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Symplectic Geometry

Homework 1

Exercise 1 (Symplectic Linear Algebra)**.** (30 points)
Exercises 1 (4 points), 2 (4 points), 3 (4 points), 4 (5 points), 5 (4 points), 6 (4 points) and 7 (5 points) from Homework 1 (on page 8) in *Lectures on Symplectic Geometry* by A. Cannas da Silva. Also available online at: http://www.mi.uni-koeln.de/∼pabiniak/sg.html

Exercise 2 (The graph of a symplectomorphism is a Lagrangian)**.** (10 points)
Let (V_1, ω_1), (V_2, ω_2) be symplectic vector spaces and let $\Psi\colon V_1 \to V_2$ be a linear isomorphism. Prove that Ψ is a symplectomorphism if and only if the graph

$$\Gamma_\Psi = \big\{(v, \Psi(v)) \mid \ v \in V_1\big\}$$

is a Lagrangian submanifold of $V_1 \times V_2$ with symplectic form $(-\omega_1) \times \omega_2$.
(This notation means that $(-\omega_1)\times\omega_2$ evaluated on the pair of vectors $((v_1, v_2), (v_1', v_2'))$ is equal to $-\omega_1(v_1, v_1') + \omega_2(v_2, v_2')$. The manifold $V_1 \times V_2$ with symplectic form $(-\omega_1)\times\omega_2$ is usually denoted by $\overline{V}_1 \times V_2$.)

Hand in: Thursday October 27th
in the first exercise session
in Übungsraum 1, MI